inertia, it posesses the same dynamic properties as the original rigid body. Routh[4] studied the mutual gravitational potential between two rigid bodies and obtained the first approximate expression for the gravitational potential for the case in which the distance between centers of mass of bodies is much larger than the size of each body. Only moments of inertia of each body are involved in the correction term of the gravitational potential. Thus, we conclude that the equimomental systems also have the same gravitational potential as the original rigid bodies up to the first approximation.

References

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[2]Whittaker, E. T., *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies,* Cambridge Univ. Press, Cambridge, England, UK, 1959, pp. 117–130.

[3]Greenwood, D. T., *Principles of Dynamics,* Prentice-Hall, Englewood Cliffs, NJ, 1965, problem 7.3 on p. 354 with solution on p. 506.

[4]Routh, E. J., *Treatise on the Dynamics of a System of Rigid Bodies, Advanced Part,* Dover, New York, 1955, pp. 340–346.

Technical Comments

Comment on "Generalized Technique for Inverse Simulation Applied to Aircraft Maneuvers"

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Introduction

HESS et al.[1] introduced a new method for inverse simulation in their paper entitled "Generalized Technique for Inverse Simulation Applied to Aircraft Maneuvers"; this method is called the integration inverse method. The method assumes that the input is constant in the discretization interval T. The algorithm starts with an initial guess of the input. A forward simulation over T follows. The variables at the end of the interval are then compared with the desired trajectory. Newton's method is then used to correct the initial guess of the input based on the Jacobian and the errors. The iterative procedure processes until the input converges. The input time history resulting from this method will be a step function in every interval T.

Analysis

Consider the linear vehicle model:

$$\dot{x} = Ax + Bu \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

Let nx be the number of states, nu the number of inputs, and ny the number of outputs. The authors of the paper[1] claimed that the method is independent of the values of nx, nu, and ny, provided $ny \leq nu$. As the following example will show, in the situation where $nx > nu$ and $nx > ny$, the method may be unstable for small T.

Consider the second-order linear system:

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = u(t) \tag{3}$$

Assume that $x(t)$ is specified by the desired trajectory $x_d(t)$, and $\dot{x}(t)$ is not specified. This represents a case that $nx = 2$ and $ny = nu = 1$. If the input $u(t) = \hat{u} = \text{const}$ over the period $t_0 \leq t \leq t_0 + T$, the solution of Eq. (3) is

$$x(t) = x_0 e^{-\sigma(t-t_0)}\left\{\cos[\omega_d(t-t_0)] + \frac{\sigma}{\omega_d}\sin[\omega_d(t-t_0)]\right\} + \frac{\dot{x}_0}{\omega_d}e^{-\sigma(t-t_0)}\sin[\omega_d(t-t_0)] + \frac{\hat{u}}{\omega_n^2}\left(1 - e^{-\sigma(t-t_0)}\left\{\cos[\omega_d(t-t_0)] - \frac{\sigma}{\omega_d}\sin[\omega_d(t-t_0)]\right\}\right) \tag{4}$$

where $\sigma = \zeta\omega_n$, $\omega_d = \omega_n\sqrt{1-\zeta^2}$, $x_0 = x(t_0)$, and $\dot{x}_0 = \dot{x}(t_0)$. The input in the interval $t_0 \leq t \leq t_0 + T$ can be solved as

$$\hat{u} = \omega_n^2\{x_1 - x_0 e^{-\sigma T}[\cos(\omega_d T) + (\sigma/\omega_d)\sin(\omega_d T)] - (\dot{x}_0/\omega_d)e^{-\sigma T}\sin(\omega_d T)\}/\{1 - e^{-\sigma T}[\cos(\omega_d T) - (\sigma/\omega_d)\sin(\omega_d T)]\} \tag{5}$$

where $x_1 = x(t_0 + T)$. If there are errors in the variables, the error of the input is

$$\Delta\hat{u} = \frac{\partial\hat{u}}{\partial x_1}\Delta x_1 + \frac{\partial\hat{u}}{\partial x_0}\Delta x_0 + \frac{\partial\hat{u}}{\partial\dot{x}_0}\Delta\dot{x}_0 \tag{6}$$

For the case that $T \ll 1$, the partial derivatives are $\partial\hat{u}/\partial x_1 \approx 2/T^2$, $\partial\hat{u}/\partial x_0 \approx -2/T^2$, and $\partial\hat{u}/\partial\dot{x}_0 \approx -2/T$.

In the ideal case, $x_1 = x_d(t_0 + T)$. However, because there are always some errors in the Newton's iteration and in the forward simulation, the term Δx_1 exists. $\Delta x_0 = 0$ if $x(t_0)$ is reset to $x_d(t_0)$ before the iteration procedure starts. The term $\Delta\dot{x}_0 \neq 0$ since the state variable $\dot{x}(t)$ is not specified. In summary, the error Δx_1 in each step is amplified by a factor $2/T^2$ then carried over to the next step through the variable $\dot{x}(t)$. The variable $\dot{x}(t)$ will drift away from the true value as time progresses. For very small T, the error grows quickly.

Example

The solution of the differential equation

$$\ddot{x}(t) + \dot{x}(t) + x(t) = \sin 2t \tag{7}$$

with initial conditions $x(0) = 0$ and $\dot{x}(0) = 0$ is

$$x(t) = 1/13\ \{e^{-0.5t}[2\cos\sqrt{0.75}t + (7/\sqrt{0.75})\sin\sqrt{0.75}t] - 2\cos 2t - 3\sin 2t\} \tag{8}$$

The inverse simulation is formulated as follows: use Eq. (8) as the desired trajectory and find the input function.

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The integration inverse method is used in this example. The convergence criterion used in the Newton-Ralphson method is the error $\leq 10^{-6}$. Euler integration is used in the forward simulation with a step size $T/1000$ s. Figure 1 shows the inverse simulation results when $T = 0.1$ s, which is very close to the exact solution sin $2t$. Figure 2 shows the results when $T = 0.01$ s. The solution exhibits the high-frequency oscillations mentioned in the paper (Ref. 1, p. 922). When T is less than 0.005 s, the result becomes erratic and unstable, as shown in Fig. 3.

Conclusion

From the preceding analysis and example, we can conclude that if there are unspecified intermediate variables in the system, the discretization interval T should not be too small. Hence the choice of T is based on two factors:

1) It has to be small enough so that the assumption $u(t) = \text{const}$ in $t_0 \leq t \leq t_0 + T$ is valid.

2) It has to be large enough so that the errors carried by the unspecified intermediate variables are insignificant.

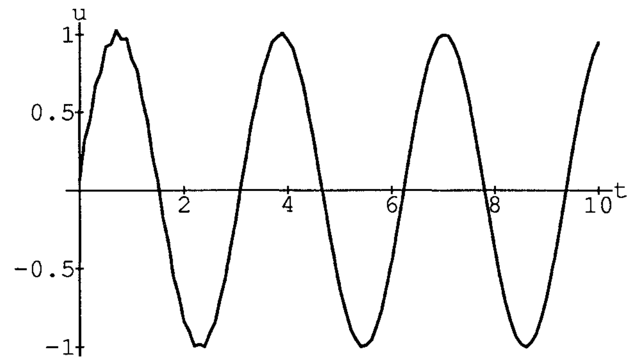

Fig. 1 Result of inverse simulation, $T = 0.1$ s.

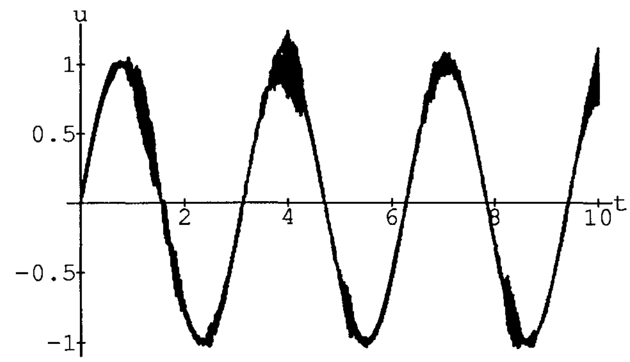

Fig. 2 Result of inverse simulation, $T = 0.01$ s.

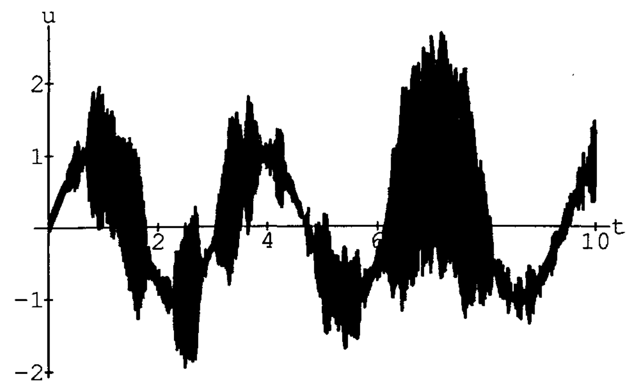

Fig. 3 Result of inverse simulation, $T = 0.005$ s.

References

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Reply by Authors to Kuo-Chi Lin

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We wish to thank Professor Lin for his interest in our paper and for his instructive example and comments. In Ref. 1, we have attempted to provide a technique for obtaining accurate representations of the control inputs that will force a nonlinear dynamic system to follow a desired trajectory with a fidelity suitable for engineering analyses. The resulting algorithm allows the number of inputs to be equal to or larger than the number of constrained (desired) outputs.

Through a nice example, Professor Lin has demonstrated a phenomenon that we discussed briefly in Ref. 1. Quoting from Ref. 1: "In some of the inverse solutions to be discussed, the control inputs obtained from the simulation exhibited low-amplitude, high-frequency oscillations superimposed on the low-frequency waveform.... These oscillations were filtered by the vehicle dynamics and had minimal impact on the solution quality. They were removed from the (input) solutions to be discussed by use of a fifth-order, low-pass digital filter in the simulation process. The filter had a cutoff frequency of 10 rad/s. The control inputs were passed through the filter twice (forward and backward) to avoid time shifting in the output data."

None of the input solutions in Ref. 1 exhibited the large-amplitude oscillations shown in Fig. 3 of Professor Lin's comment. However, we did obtain oscillations with relatively large amplitudes in a more recent application of the inverse simulation technique.[2] In spite of these oscillations, the filtering technique just described, while admittedly inelegant, allowed us to meet the objective stated in the first paragraph.

In the Discussion section of Ref. 2 we state, "It would be desirable to minimize the oscillatory control inputs which sometimes occur in the inverse simulation. Algorithm modifications which may alleviate this problem are 1) allowing the discretization interval T to be adaptive with respect to the value of the error function $F_E, \ldots$: and 2) improving the accuracy of the Jacobian matrix by perturbing the system with multiple inputs Δu, both positive and negative in sign, and using the average values of the various resulting $\partial y_i/\partial u_j$ as elements of the Jacobian J."

Rather than attempting to adjust T in each of our inverse solutions, we have demonstrated that filtering the control inputs has allowed us to approximate the "true" control inputs with sufficient accuracy for engineering analyses.[1-3] We hasten to point out that, in Refs. 1–3, we always demonstrate the quality of the resulting control input solution by using the filtered control signals as inputs to a forward simulation of the dynamic system in question.

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